MAT 168 Calculus II TEST 1

1. Find the antiderivative F of $f(x) = 4 - 3(1+x^2)^{-1}$ that satisfies $F(1) = -3$.

   Solution:

$$\begin{aligned} F(x) &= \int \left(4 - \frac{3}{1+x^2}\right) dx \\ &= 4x - 3\tan^{-1} x + C \end{aligned}$$

   We require that

$$F(1) = 4(1) - 3\tan^{-1}(1) = 4 - \frac{3\pi}{4} + C = -3$$

   So, we obtain $C = -7 + \frac{3\pi}{4}$ and hence

$$F(x) = 4x - 3\tan^{-1} x - 7 + \frac{3\pi}{4}$$

2. Find f if $f''(x) = 3\sin x$, $f'(\pi) = 5$, $f(\pi) = -1$.

   Solution: $f'(x) = -3\cos x + C_1$ and $f(x) = -3\sin x + C_1 x + C_2$. From $f'(\pi) = 5$, we have

$$-3(-1) + C_1 = 3 + C_1 = 5$$

   and so $C_1 = 2$. From $f(\pi) = -1$, we have

$$-3(0) + 2(\pi) + C_2 = 2\pi + C_2 = -1$$

   and so $C_2 = -1 - 2\pi$. Therefore,

$$f(x) = -3\sin x + 2x - 1 - 2\pi$$

3. Find the function $f(x)$ described b ythe given intial value problem.

$$f''(x) = 7\sin x,\ f'(\pi) = -3,\ f(\pi) = -2$$

Solution: $f'(x) = -7\cos x + C_1$. From $f'(\pi) = -3$, we have $7 + C_1 = -3$ that is $C_1 = -10$. Now $f(x) = -7\sin x - 10x + C_2$. From $f(\pi) = -2$, we have $-10\pi + C_2 = -2$ that is $C_2 = 10\pi - 2$. Therefore

$$f(x) = -7\sin x - 10x + 10\pi - 2$$

4. Find the indefinite integral

$$\int \left(4x^2 + 10 + \frac{8}{x^2+1}\right) dx$$

Solution:

$$\begin{aligned}\int \left(4x^2 + 10 + \frac{8}{x^2+1}\right) dx &= 4\int x^2 dx + 10\int dx + 8\int \frac{dx}{x^2+1} \\ &= \frac{4}{3}x^3 + 10x + 8\tan^{-1} x + C\end{aligned}$$

5. Use the Fundamental Theorem of Calculus to find the derivative of

$$F(x) = \int_x^{10} \tan(t^2) dt$$

Solution:

$$\begin{aligned}F'(x) &= \frac{d}{dx}\int_x^{10} \tan(t^2) dt \\ &= -\frac{d}{dx}\int_{10}^{x} \tan(t^2) dt \\ &= -\tan(x^2)\end{aligned}$$

6. Use the Fundamental Theorem of Calculus to find the drivative of

$$y = \int_{-2}^{\sqrt{x}} \frac{\cos t}{t^5} dt$$

Solution: Since y is a composite function we need to use the chain rule along with the Fundamental Theorem of Calculus to find y'. Let $u = \sqrt{x}$. Then $y = \int_1^u (9t + 10\sin t)dt$ and $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$.

$$\begin{aligned}
y' &= \frac{d}{du}\int_1^u (9t + 10\sin t)dt\frac{du}{dx} \\
&= \frac{\cos\sqrt{x}}{(\sqrt{x})^5}\frac{1}{2\sqrt{x}} \\
&= \frac{\cos\sqrt{x}}{2(\sqrt{x})^6} \\
&= \frac{\cos\sqrt{x}}{2x^3}
\end{aligned}$$

7. Evaluate the definite integral

$$\int_1^6 \frac{6x^2+9}{\sqrt{x}}dx$$

Solution:

$$\begin{aligned}
\int_1^6 \frac{6x^2+9}{\sqrt{x}}dx &= \int_1^6 \frac{6x^2+9}{x^{\frac{1}{2}}}dx \\
&= 6\int_1^6 \frac{x^2}{x^{\frac{1}{2}}} + 9\int_1^6 \frac{1}{x^{\frac{1}{2}}} \\
&= 6\int_1^6 x^{\frac{3}{2}}dx + 9\int_1^6 x^{-\frac{1}{2}}dx \\
&= 6\left(\frac{2}{5}\right)[x^{\frac{5}{2}}]_1^6 + \frac{9}{2}[x^{\frac{1}{2}}]_1^6 \\
&= \frac{87(6)^{\frac{3}{2}} - 102}{5}
\end{aligned}$$

8. Evaluate the integral

$$\int_1^{\sqrt{3}} \frac{4}{1+x^2}dx$$

Solution:

$$\int_1^{\sqrt{3}} \frac{4}{1+x^2} dx = 4[\tan^{-1} x]_1^{\sqrt{3}}$$
$$= 4\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$
$$= \frac{\pi}{3}$$

9. Evaluate the integral

$$\int_0^4 (4e^x + 6\cos x) dx$$

Solution:

$$\int_0^4 (4e^x + 6\cos x) dx = 4[e^x]_0^4 + 6[\sin x]_0^4$$
$$= 4(e^4 - 1) + 6\sin 4$$

10. Evaluate the indefinite integral

$$\int \frac{-8}{x(\ln x)^2}$$

Solution: Let $u = \ln x$. Then $du = \frac{1}{x} dx$. So,

$$\int \frac{-8}{x(\ln x)^2} = -8 \int \frac{du}{x^2}$$
$$= \frac{8}{u} + C$$
$$= \frac{8}{\ln x} + C$$

11. Evaluate

$$\int_0^{\pi/3} \tan x dx$$

Solution: Let $u = \cos x$. Then $du = -\sin x\,dx$ and $u(0) = 1$, $u(\pi/3) = \frac{1}{2}$. Hence,

$$\begin{aligned}\int_0^{\pi/3} \tan x\,dx &= -\int_1^{1/2} \frac{du}{u} \\ &= -[\ln u]_1^{1/2} \\ &= \ln 2\end{aligned}$$

12. Evaluate the indefinite integral

$$\int 4e^{4x} \sin(e^{4x})dx$$

Solution: Let $u = e^{4x}$. Then $du = 4e^{4x}dx$. Hence,

$$\begin{aligned}\int 4e^{4x} \sin(e^{4x})dx &= \int \sin u\,du \\ &= -\cos u + C \\ &= -\cos(e^{4x}) + C\end{aligned}$$

13. Evaluate the generla antiderivative $F(x)$ of the function

$$f(x) = 7x\sin(x^2)$$

Solution: Let $u = x^2$. Then $du = 2x\,dx$. Hence,

$$\begin{aligned}F(x) &= \int 7x\sin(x^2)dx \\ &= \frac{7}{2}\int \sin u\,du \\ &= -\frac{7}{2}\cos u + C \\ &= -\frac{7}{2}\cos x^2 + C\end{aligned}$$