MAT 168 Calculus II TEST 3

1. Use integration by parts to evaluate the definite integral

$$\int_0^1 te^{-t}dt$$

Solution: We use tabular integration. In the following table, the first column represents t and its derivatives, and the second column represents e^{-t} and its integrals.

$$\begin{array}{ccc} t & & e^{-t} \\ & \overset{+}{\searrow} & \\ 1 & & -e^{-t} \\ & \overset{-}{\searrow} & \\ 0 & & e^{-t} \end{array}$$

Following this table, we obtain

$$\int_0^1 te^{-t}dt = [-te^{-t} - e^{-t}]_0^1 = -2e^{-1} + 1$$

2. Use integration by parts to evaluate the integral

$$\int 8x^2 \cos 2x dx$$

Solution: We use tabular integration. In the following table, the first column represents x^2 and its derivatives, and the second column represents $\cos 2x$ and its integrals.

x^2		$\cos 2x$
	$\overset{+}{\searrow}$	
$2x$		$\frac{1}{2}\sin 2x$
	$\overset{-}{\searrow}$	
2		$-\frac{1}{4}\cos 2x$
	$\overset{+}{\searrow}$	
0		$-\frac{1}{8}\sin 2x$

From this table, we obtain

$$\int x^2 \cos 2x dx = \frac{x^2}{2}\sin 2x + \frac{x}{2}\cos 2x - \frac{1}{4}\sin 2x$$

where the constant of integration is neglected for the moment. Therefore,

$$\int 8x^2 \cos 2x dx = 4x^2 \sin 2x + 4x\cos 2x - 2\sin 2x + C$$

3. Find the integral

$$\int e^{2x} \sin 2x dx$$

Solution: We use tabular integration. In the following table, the first column represents $\sin 2x$ and its derivatives, and the second column represents e^{2x} and its integrals.

$\sin 2x$		e^{2x}
	$\overset{+}{\searrow}$	
$2\cos 2x$		$\frac{1}{2}e^{2x}$
	$\overset{-}{\searrow}$	
$-4\sin 2x$		$\frac{1}{4}e^{2x}$

From this table, we obtain

$$\int e^{2x}\sin 2x dx = \frac{1}{2}e^{2x}\sin 2x - \frac{1}{2}e^{2x}\cos 2x - \int e^{2x}\sin(2x)dx + C_1$$

Therefore,

$$\int e^{2x} \sin 2x dx = \frac{1}{4} e^{2x} \sin 2x - \frac{1}{4} e^{2x} \cos 2x + C$$

where $C = \frac{C_1}{2}$.

4. Evaluate the integral

$$\int -\sin^3 x \cos^5 x dx$$

Solution:

$$\begin{aligned}
\int -\sin^3 x \cos^5 x dx &= -\int \sin^3 x \cos^4 x \cos x dx \\
&= -\int \sin^3 x (\cos^2 x)^2 \cos x dx \\
&= -\int \sin^3 x (1 - \sin^2 x)^2 \cos x dx \\
&= -\int u^3 (1 - u^2)^2 du \; (u = \sin x) \\
&= \int (-u^3 + 2u^5 - u^7) du \\
&= -\frac{u^4}{4} + \frac{u^6}{3} - \frac{u^8}{8} + C \\
&= -\frac{\sin^4 x}{4} + \frac{\sin^6 x}{3} - \frac{\sin^8 x}{8} + C
\end{aligned}$$

5. Evaluate the integral

$$\int -\tan^3 x dx$$

Solution:

$$
\begin{aligned}
\int -\tan^3 x dx &= -\int \tan x \tan^2 x dx \\
&= -\int \tan x(\sec^2 x - 1)dx \\
&= -\int \tan x \sec^2 x dx + \int \tan x dx \\
&= -\int u du - \int \frac{dv}{v} \ (u = \tan x,\ v = \cos x) \\
&= -\frac{u^2}{2} - \ln v + C \\
&= -\frac{\tan^2 x}{2} - \ln(\cos x) + C
\end{aligned}
$$

6. Evaluate the integral

$$\int 2\cos^2(4x)dx$$

Solution: From half-angle formula we have $\cos^2 4x = \frac{1+\cos 8x}{2}$. Thus

$$
\begin{aligned}
\int 2\cos^2(4x)dx &= \int dx + \int \cos 8x dx \\
&= x + \frac{\sin 8x}{8} + C
\end{aligned}
$$

7. Evaluate the integral using the indicated trigonometric substitution.

$$\int \frac{-x^3}{\sqrt{x^2+9}} dx,\ x = 3\tan\theta$$

Solution: Since $x = 3\tan\theta$, $x^2 + 9 = 9(\tan^2\theta + 1) = 9\sec^2\theta$ and

$dx = 3\sec^2\theta d\theta$. Thus

$$\begin{aligned}
\int \frac{-x^3}{\sqrt{x^2+9}}dx &= -27\int \tan^3\theta \sec\theta d\theta \\
&= -27\int \tan^2\theta \tan\theta \sec\theta d\theta \\
&= -27\int (\sec^2\theta - 1)\sec\theta \tan\theta d\theta \\
&= -27\int (u^2-1)du \ (u = \sec\theta) \\
&= -9u^3 + 27u + C \\
&= -9\sec^3\theta + 27\sec\theta + C
\end{aligned}$$

Since $\tan\theta = \frac{x}{3}$, $\sec\theta = \frac{\sqrt{x^2+9}}{3}$. Therefore,

$$\int \frac{-x^3}{\sqrt{x^2+9}}dx = -\frac{(x^2+9)\sqrt{x^2+9}}{3} + 9\sqrt{x^2+9} + C$$

8. Evaluate the indefinite integral.

$$\int \frac{\sqrt{x^2-4}}{x}dx$$

Solution: Let $x = 2\sec\theta$. Then $x^2 - 4 = 4(\sec^2\theta - 1) = 4\tan^2\theta$ and $dx = 2\sec\theta\tan\theta$. Thus

$$\begin{aligned}
\int \frac{\sqrt{x^2-4}}{x}dx &= 2\int \tan^2\theta d\theta \\
&= 2\int (\sec^2\theta - 1)d\theta \\
&= 2\tan\theta - 2\theta + C \\
&= \sqrt{x^2-4} - 2\sec^{-1}\left(\frac{x}{2}\right) + C
\end{aligned}$$

9. Evaluate the indefinite integral.

$$\int \frac{1}{x^2-2x-3}dx$$

Solution: $x^2 - 2x - 3 = (x-3)(x+1)$ so let

$$\frac{1}{x^2-2x-3} = \frac{A}{x-3} + \frac{B}{x+1}$$

Then we have

$$1 = A(x+1) + B(x-3) = (A+B)x + A - 3B$$

Comparing the coefficients we obatin the equation $A + B = 0$ and $A - 3B = 1$. Solve the equations simulataneously to find $A = \frac{1}{4}$ and $B = -\frac{1}{4}$. Therefore,

$$\begin{aligned}\int \frac{1}{x^2-2x-3}dx &= \frac{1}{4}\int \frac{dx}{x-3} - \frac{1}{4}\int \frac{dx}{x+1} \\ &= \frac{1}{4}\ln|x-3| - \frac{1}{4}\ln|x+1| + C\end{aligned}$$

10. Evaluate the integral.

$$\int_0^1 \frac{3x+7}{x^2+5x+6}dx$$

Solution: $x^2 + 5x + 6 = (x+2)(x+3)$ so let

$$\frac{3x+7}{x^2+5x+6} = \frac{A}{x+2} + \frac{B}{x+3}$$

Then we have

$$3x + 7 = A(x+3) + B(x+2) = (A+B)x + 3A + 2B$$

Comparing the coefficients we obtain the equations $A + B = 3$ and $3A + 2B = 7$. Sove the equations simulataneously to find $A = 1$ and $B = 2$. Therefore,

$$\begin{aligned}\int_0^1 \frac{3x+7}{x^2+5x+6}dx &= \int_0^1 \frac{dx}{x+2} + 2\int_0^1 \frac{dx}{x+3} \\ &= [\ln|x+2|]_0^1 + 2[\ln|x+3|]_0^1 \\ &= -\ln 3 + 3\ln 2\end{aligned}$$