

MAT 421 Number Theory

Takehome Exam 1

Note: Read the test instructions in my email carefully and thoroughly before you begin your exam. Failure to follow the instructions could result in point reductions or no point on individual problems.

1. Find $(165, 465)$ using the Euclidean algorithm. Then solve the equation

$$165x + 465y = (165, 465)$$

by going backward of the Euclidean algorithm from bottom to top.

2. Prove: Given integers a , b , and c with a and b not both 0, there exist $x, y \in \mathbb{Z}$ such that $ax + by = c$ if and only if $(a, b) | c$.
3. Use the Euclidean algorithm to find one solution to $66x + 51y = 300$.
4. Describe all solutions of $3x \equiv 4 \pmod{7}$.
5. Find the smallest nonnegative solution of the system of congruences:

$$x \equiv 2 \pmod{3}$$

$$x \equiv 3 \pmod{5}$$

$$x \equiv 4 \pmod{11}$$

$$x \equiv 5 \pmod{16}$$