MAT 421 Number Theory
Takehome Exam 2

Note: Read the test instructions in my email carefully and thoroughly before you begin your exam. Failure to follow the instructions could result in point reductions or no point on individual problems.

1. In the 27-letter alphabet (with blank=26), use the affine enciphering transformation with key $a = 13$, $b = 9$ to encipher the message "HELP ME."

2. In a long string of ciphertext which was encrypted by means of an affine map on single-letter message units in the 26-letter alphabet, you observe that the most frequently occurring letters are "Y" and "V", in that order. Assuming that those ciphertext message units are the encryption of "E" and "T", respectively, read the message "QAOOYQQEVHEQV".

3. Find the inverse matrix of

$$\begin{pmatrix} 15 & 17 \\ 4 & 9 \end{pmatrix} \mod 26$$

   Write the entries in the inverse matrix as nonnegative integers less than 26.

4. Find all solutions $\begin{pmatrix} x \\ y \end{pmatrix} \mod N$, writing x and y as nonnegative integers less than N.

   (a)

$$\begin{aligned} x + 4y &\equiv 1 \mod 9 \\ 5x + 7y &\equiv 1 \mod 9 \end{aligned}$$

(b)

$$\begin{aligned} x + 4y &\equiv 1 \mod 9 \\ 5x + 8y &\equiv 2 \mod 9 \end{aligned}$$

5. You intercepted the message "SONAFQCHMWPTVEVY", which you know resulted from a linear enciphering transformation of digraph-vectors, where the sender used the usual 26-letter alphabet A-Z with numerical equivalents 0-25, respectively An earlier statistical analysis of a long string of intercepted ciphertext revealed that the most frequently occurring ciphertext digraphs were "KH" and "XW" in that order. You take a guess that those digraphs correspond to "TH" and "HE", respectively, since those are the most frequently occurring digraphs in most long plaintext messages on the subject you think is being discussed. Find the deciphering matrix, and read the message.