MAT 421 Number Theory
Takehome Final Exam

Note: Read the test instructions in my email carefully and thoroughly before you begin your exam. Failure to follow the instructions could result in point reductions or no point on individual problems.

1. Find $(217, 341)$ using the Euclidean algorithm. Then solve the equation
$$217x + 341y = (217, 341)$$
by going backward of the Euclidean algorithm from bottom to top.

2. Find all solutions of $6x + 8y = 120$ with x and y positive.

3. Find the smallest nonnegative solution of the system of congruences
$$\begin{aligned} 19x &\equiv 103 \mod 900 \\ 10x &\equiv 511 \mod 841 \end{aligned}$$

4. Use the repeated squaring method to find $38^{75} \mod 103$.

5. Use the Lucas-Lehmer test to determine if $M_{13} = 2^{13} - 1$ is a prime.