Coarse-scale Modeling of Flow in Gas-injection Processes for Enhanced Oil Recovery

James V. Lambers[1]

Department of Energy Resources Engineering, Stanford University, 367 Panama St Rm 094, Stanford, CA 94305-2220 lambers@stanford.edu

1 Introduction

Subsurface formations that arise in the simulation of gas-injection processes for enhanced oil recovery may exhibit geometrically complex features with complicated large-scale connectivity. They must be included in simulations of flow and transport because they can fundamentally impact simulation results. However, to reduce computational costs, simulations are generally performed on grids that are coarse compared to the given geocellular grids, so accurate upscaling is required.

In this work we are concerned with transmissibility upscaling. In the presence of full-tensor effects, multi-point flux approximations (MPFA) are desirable for the sake of accuracy, as opposed to two-point flux approximations (TPFA). However, MPFA methods add computational costs and may suffer from non-monotonicity (see [NAE06]).

In [LGM06], Variable Compact Multi-Point (VCMP) upscaling was introduced. This method constructs a local MPFA that accommodates full-tensor anisotropy, and guarantees a monotone pressure solution (see [GLM06]). While it has been demonstrated that VCMP performs quite well, compared to other upscaling methods, it does not perform as well for highly channelized domains that are likely to arise in the simulation of gas injection processes.

In this paper, we consider some modifications to VCMP in order to improve its accuracy for such cases. The general design of the VCMP methods is summarized in section 2. In Section 3, we present strategies for improving the accuracy and/or efficiency of VCMP. We discuss the results and future directions in section 4.

2 Variable Compact Multi-Point (VCMP) Upscaling

In this section, we briefly review VCMP upscaling. We consider single phase, steady and incompressible flow in a heterogeneous reservoir. The governing dimensionless pressure equation, at the fine and coarse scales, is

$$\nabla \cdot (k \cdot \nabla p) = 0. \tag{1}$$

Here p is the pressure and k the permeability tensor, all of which are non-dimensionalized by appropriate reference values.

For simplicity, we consider a two-dimensional reservoir, and describe how VCMP upscales transmissibility to a Cartesian coarse grid. To create a MPFA, we allow the stencil to vary per cell face. Our MPFA uses a subset of the six pressure values p_j, $j = 1, \cdots, 6$, at the six coarse cell centers nearest the face, where $j = 1$ and $j = 2$ correspond to the points that would be used in a TPFA. For each j, we let t_j denote the weight that will be assigned to point j in the flux approximation, which has the general form $f = -\mathbf{t}^T\mathbf{p}$, where $t = \left[\, t_1 \cdots t_6 \,\right]^T$, $p = \left[\, p_1 \cdots p_6 \,\right]^T$.

We solve the pressure equation on a local region of the fine grid containing the six points with two sets of generic boundary conditions. We let $\mathbf{p}^1(x, y)$ and $\mathbf{p}^2(x, y)$ be the solutions of these local problems, and p_j^i denote the value of $\mathbf{p}^i(x, y)$ at point j. For $i = 1, 2$, we let f_i denote the coarse-scale flux across the face obtained from the local solution $\mathbf{p}^i(x, y)$. To compute the weights t_j, we solve the general optimization problem

$$\min_t \sum_{i=1}^{2} \alpha_i^2 |\mathbf{t}^T\mathbf{p}^i - f_i|^2 + \sum_{j=3}^{6} \beta_j^2 t_j^2, \tag{2}$$

subject to the essential linear constraints to maximize robustness. Extension to quasi-Cartesian adapted grids is discussed in [LGM06].

3 Modifications to VCMP

In this section, we consider three strategies for improving the accuracy and efficiency of VCMP, particularly for channelized domains.

3.1 Combination with MLLG Upscaling

Local-global (LG) upscaling, introduced in [CDGW03], offers improved accuracy for permeability fields that exhibit strong global connectivity. Combined with local grid adaptation strategies, LG helps to reduce process dependency and leads to improved efficiency. This combination is known as Multi-Level Local-global upscaling, introduced in [GL06].

The distinction between local-global methods and local methods, such as VCMP, is that local-global methods compute global solutions of the pressure equation (1) on the coarse grid. Then, these global solutions are interpolated at points on the boundary of each extended local region. These boundary values serve as Dirichlet data for the local fine-scale solves that are used to compute upscaled transmissibilities. An iteration is used to ensure consistency between the fine and coarse scales, and because the boundary data for the local solves can account for global connectivity, greater accuracy is achieved for highly channelized domains than for local methods, including VCMP.

Like any other local method, VCMP is easily modified to use a local-global approach. However, the interpolation of the global coarse-scale solutions to the fine grid is a crucial step that must be performed carefully to ensure the same accuracy and robustness that VCMP can deliver for other domains. Whereas MLLG only uses linear interpolation, we consider both quadratic and cubic spline interpolation. Preliminary results clearly demonstrate substantial (up to 50%) improvement in the accuracy of the coarse-scale resolution of the fine-scale velocity field for channelized domains, compared to using the original, local VCMP method or MLLG with linear interpolation.

3.2 Criteria for Adaptive Mesh Refinement

MLLG includes a scheme for adaptive mesh refinement in which cells in high-flow regions are refined isotropically (for details see [GL06]). In addition to this criteria, a local-global version of VCMP will refine around faces for which it is unable to compute weights t_i with the proper sign based on local flow, or the computed MPFA causes the matrix for the global pressure solve to have an off-diagonal element with an incorrect sign. To determine which cells lie in high-flow regions, we compute the fluxes across each face in the coarse grid from global pressure fields, and compare them to the total flow. If the magnitude of the flux across the face is considered to be too high, then cells surrounding the face are refined.

However, this refinement may not be necessary if the high flow occurs within a channel that is nearly orthogonal to the face. We therefore use a simple channel-detection scheme in which refinement is not performed when flow across a face is nearly equal to flow across neighboring faces with the same orientation, and flow across neighboring faces with other orientations is negligible. Experimentation has demonstrated that such a channel detection scheme allows for the same accuracy to be achieved with a modest (∼8%) reduction in the number of cells.

3.3 Anisotropic Refinement

In the interest of reducing the number of cells in the coarse grid, we consider whether it is possible, in at least some cases, to refine anisotropically without sacrificing accuracy or robustness. Initial experimentation has shown that at least a small reduction in the number of cells can be achieved, without loss of accuracy, provided that

- The aspect ratio of newly created cells is limited,
- Cells in high-flow regions are still refined isotropically,

- Cells in low-flow regions, that are only refined to improve robustness, are refined anisotropically, parallel to faces that are flagged for refinement.

4 Conclusions

We have explored three avenues of improvement in the accuracy and efficiency of a proposed combination of two new methods, VCMP and MLLG, of transmissibility upscaling for coarse-scale modeling of single-phase flow in highly heterogeneous subsurface formations. In experiments with various channelized domains, all three strategies, to varying extents, yielded improved accuracy and/or efficiency in terms of reduction of the number of cells in the coarse grid or more accurate resolution of the fine-scale velocity field. In combination, these modifications should enhance performance even further.

In future work, we will consider the use of essentially non-oscillatory (ENO) interpolation schemes in computing Dirichlet boundary data for local solves, so that the monotonicity of the pressure field is not lost in the transition from the coarse scale to the fine scale. In addition, we will develop more sophisticated tests for detecting the presence and orientation of channels in order to guide adaptivity. Finally, the various adaptivity criteria include parameters that must be tuned in order to achieve optimal performance for a given permeability field; methods for automatically setting these parameters need to be developed.

In conclusion, it can be seen that substantial progress is being made toward creating a method for automatically generating coarse-scale models for gas-injection processes that are both accurate and robust for a wide variety of permeability fields and boundary conditions.

References

[CDGW03] Chen, Y., Durlofsky, L.J., Gerritsen, M.G., Wen, X.H.: A coupled local-global upscaling approach for simulating flow in highly heterogeneous formations. Adv. Water Res. **26**, 1041–1060 (2003)

[GL06] Gerritsen, M.G., Lambers, J.V.: Integration of Local-global Upscaling and Grid Adaptivity for Simulation of Subsurface Flow in Heterogeneous Formations. Comp. Geo., in review

[GLM06] Gerritsen, M.G., Lambers, J.V., Mallison, B.T.: A Variable and Compact MPFA for Transmissibility Upscaling with Guaranteed Monotonicity. Proccedings of the 10th European Conference on the Mathematics of Oil Recovery (2006), to appear

[LGM06] Lambers, J.V., Gerritsen, M.G., Mallison, B.T.: Accurate Local Upscaling with Variable Compact Multi-point Transmissibility Calculations. Comp. Geo., Special Issue on Multiscale Methods for Flow and Transport in Heterogeneous Porous Media, to appear (2007)

[NAE06] Nordbotten, J.M., Aavatsmark, I., Eigestad, G.T.: Monotonicity of Control Volume Methods. Numer. Math. **106**(2), 255–288 (2007)